I. Solution by J. SCHEFFER, A. M., Hagerstown, Md.; ELMER SCHUYLER, M. Sc., Reading, Pa.; LON C. WALKER, Palo Alto, Cal.; H. C. WHITAKER, Ph. D., Philadelphia, Pa.; and the PROPOSER.

Take the given line as the axis of x, the line through the given point and at right angles to the given line as the y-axis, and denote the given point as $(0, y_1)$.

The required circle being of the form $x^2+y^2+2gx+2fy+c=0$....(1), and touching $y=0$....(2), $x^2+2gx+c=0$....(3), and $c=g^2$....(4).

Also, passing through $(0, y_1)$, $y_1^2+2fy_1+c=0$....(5), and this with (4) gives $2f=-\dfrac{g^2+y_1^2}{y_1}$....(6).

(1) now is $x^2+y^2+2gx-\dfrac{g^2+y_1^2}{y_1}y+g^2=0$....(7).

If (x', y') be the center of (7), $x'=-g$, $y'=\dfrac{g^2+y_1^2}{2y_1}$, and eliminating g from these two equations, $x'^2=2y_1(y'-\frac{1}{2}y_1)$, a common parabola.

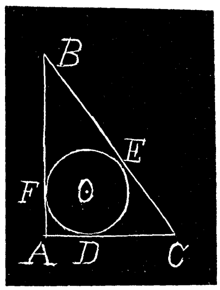
II. Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.; P. S. BERG, B. Sc., Larimore, N. D.; and H. R. HIGLEY, East Stroudsburg, Pa.

Since the distance of the center from the given straight line is always equal to its distance from the given point, both being equal to the radius of the circle, the locus of the center is a parabola having the given straight line for directrix and the given point for focus.

ANOTHER PROOF OF THE PYTHAGOREAN THEOREM.

By E. S. LOOMIS, Ph. D., Teacher of Mathematics, West High School, Cleveland, Ohio.

Let ABC be a right triangle whose sides are tangent to the circle O. Since $CD=CF$, $BF=BE$, and $AE=AD=r=$radius of circle, it is easily shown that $(CB=a)+2r=(AC+AB=b+c)$. And if $a+2r=b+c$....(1), then $(1)^2=(2)$ $a^2+4ar+4r^2=b^2+2bc+c^2$. Now if $4ar+4r^2=2bc$, then $a^2=b^2+c^2$. But $4ar+4r^2$ is greater than, equal to, or less than $2bc$.



If $4ar+4r^2>$ or $<2bc$, then $a^2+4ar+4r^2>$ or $<b^2+2bc+c^2$; *i. e.* $a+2r<$ or $>b+c$, which is absurd.

$\therefore 4ar+4r^2=2bc$.

$\therefore a^2=b^2+c^2$. Q. E. D.

NOTE. So far as we know, this proof has not been given before. If it has not been published before, it may be properly called a *new proof*. Dr. Loomis asks if any one can derive, by this method, a direct proof—the one above being indirect. ED. F.

CALCULUS.

116. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

"Prove that the length of the *greatest* beam of square section that can be cut from a log l feet long and in the shape of a conic frustum, diameters D and d, is $\frac{1}{3}lD\div(D-d)$ feet.'

Solution by C. HORNUNG, A. M., Heidelberg University, Tiffin, Ohio, and G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.

The altitude of the cone of which the given frustum is a part is easily found to be $Dl\div(D-d)$.

Let x=length of the required beam; then the diameter of the circle circumscribing the end of the beam is found to be $\dfrac{Dl-(D-d)x}{l}$, and the area of the end or section$=\dfrac{[Dl-(D-d)x]^2}{2l^2}$.

$\therefore$ the volume of the beam, $V=\dfrac{[Dl-(D-d)x]^2x}{2l^2}$.

Placing the first derivative of V equal to zero, and solving for x we have

$$x=\tfrac{1}{3}lD\div(D-d), \text{ or } lD\div(D-d).$$

The first of these values of x renders V a maximum, which was to be proved.

Also solved by *J. SCHEFFER*, and *W. O. PRUITT*.

117. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

A frustum of a paraboloid of revolution closed at both ends has a given volume. Find its interior dimensions when its surface is a minimum.

Solution by the PROPOSER.

Let $y^2=4ax$ be the equation to the parabola, c=capacity of frustum.

$\therefore \frac{8}{3}\pi\sqrt{a}[(x_2+a)^{\frac{3}{2}}-(x_1+a)^{\frac{3}{2}}]+4\pi a(x_2+x_1)=u=\text{minimum}.$

$2\pi a(x_2^2-x_1^2)=c\ldots.(2).$

$\therefore \dfrac{dx_2}{dx_1}=\dfrac{(x_1+a)^{\frac{1}{2}}-\sqrt{a}}{(x_2+a)^{\frac{1}{2}}+\sqrt{a}}\ldots.(3)$, from (1).

$\dfrac{dx_2}{dx_1}=\dfrac{x_1}{x_2}\ldots.(4)$, from (2).

$$\frac{dx_2}{da}=-\frac{(x_2+a)^{\frac{3}{2}}-(x_1+a)^{\frac{3}{2}}+3a(x_2+a)^{\frac{1}{2}}-3a(x_1+a)^{\frac{1}{2}}+3\sqrt{a}(x_2+x_1)}{3a(x_2+a)^{\frac{1}{2}}+3a\sqrt{a}}\ldots(5).$$

$\dfrac{dx_2}{da}=-\dfrac{x_2^2-x_1^2}{2ax_2}\ldots.(6).$

From (3) and (4), $x_1=\dfrac{x_2\{x_2-2a-2\sqrt{[(x_2+a]a}\}}{[\sqrt{(x_2+a)}+\sqrt{a}]^2}$, or $x_1=0$.

Eliminating dx_2/da between (5) and (6) and substituting $x_1=0$, we get

$$(8a-x_2)\sqrt{(x_2+a)}=(8a+3x_2)\sqrt{a}.$$

$\therefore x_2=24a\ldots.(7)$. From (2) and (7),

$$x_2=\left(\frac{12c}{\pi}\right)^{\frac{1}{3}},\ a=\tfrac{1}{24}\left(\frac{12c}{\pi}\right)^{\frac{1}{3}}.$$

Also solved by *J. SCHEFFER*.